A Methodology for Comparing the Range Performance of Chemically Fueled Submersibles

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This paper presents a simple, general method for readily comparing the performances of chemically powered submersibles. Particularly useful in the preliminary design phase, the methodology permits quick assessment of the effects on vehicle range of variations in vehicle, engine, and reactant parameters. Criteria are given for determining when the system is weight or volume limited. The application of the methodology is illustrated for a submarine displacing 10,000 ft^3, showing the extended cruise range of a dual-engine system with one engine designed for high speed, the other for low speed. The sensitivities of the cruise and dash ranges to variations in design parameters are illustrated. The paper also discusses the effects of various system parameters on the criteria determining whether the vehicle is weight or volume limited for vehicles displacing 10,000 and 20 ft^3. This paper is based on research performed by The Rand Corporation for the Defense Advanced Research Projects Agency.

Nomenclature

a $= 1 - b$
b $= V_e^*/\theta_0 V^*$
C_d = vehicle drag coefficient based on $V^{2/3}$
f = weight of reactants plus tanks and insulation divided by weight of reactants, lb
g = gravitational constant, 32.2 ft/sec^2
h = heating value of reactants, Btu/lb
P = drag power of submersible, hp
P_c = drag power of submersible at cruise speed, hp
P_d = drag power of submersible at dash speed, hp
P_h = hotel power, hp
P_s = shaft power, hp
SRC = specific reactant consumption of engine, lb/shaft hphr
t_c = duration of cruise sortie with all reactants expended at cruise speed, hr
t_d = duration of dash sortie with all reactants expended at dash speed, hr
U = submarine speed, kn
U_c = cruise speed, kn
U_d = dash speed, kn
V = volume of submersible, ft^3
V_e $= W_e/\rho_e$, ft^3
V_r = volume of reactants and all related space, ft^3
V^* $= V/P_d$, ft^3/hp
V_e^* $= V_e/P_d$, ft^3/hp
W = gross weight of submersible, lb
W_e = weight of engine, including all propulsion-related equipment and electric generators, lb
W_r = weight of reactants times f, lb
W_e^* $= W_e/P_d$, lb/hp
W_{rc}^* = rate of reactant consumption at cruise speed times f divided by P_c, lb/hphr
W_{rd}^* = rate of reactant consumption at dash speed times f divided by P_d, lb/hphr
X_c = distance travelled if all reactants are expanded at cruise speed, naut miles
X_d = distance travelled if all reactants are expended at dash speed, naut miles

Greek Symbols

η_e = efficiency of hotel-load power generator
η_p = propeller efficiency
η_t = efficiency of transmission of power from engine shaft to propeller
η_{en} = engine efficiency, shaft output/heat input to working fluid
θ = fraction of vehicle volume available for propulsion system and reactants, including space allowed for servicing
ρ_e = weight of engine, including all propulsion-related equipment and electric generators, divided by total volume preempted by engine, including volume allowed for operation and maintenance of engine
ρ_r = weight of reactants and all related items (reactant tanks, insulation, and product tanks‡) divided by total volume preempted by reactants and all related items (reactant tanks, insulation, and product tanks‡) and volume allowed for servicing
ρ_w = density of sea water, 64 lb/ft^3
ω = fraction of W available for propulsion system and reactants, including tanks and other propulsion system components

Subscripts

c = cruise condition
d = dash condition
0 = reference conditions

I. Introduction

THIS paper presents a simple, general method for readily comparing the performances of chemically powered submersibles. The method is particularly useful in the preliminary design phase, as a means of evaluating the expected performance of competitive designs. Section II derives equations for determining the range of the submersible, and whether it is weight or volume limited as a function of the major vehicle and engine parameters. Section III demonstrates the application of the methodology. The effects of changes in the major vehicle and engine parameters on cruise and dash ranges are illustrated for submersibles displacing about 10,000 ft^3. Section III also briefly discusses the range

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‡When the reactant products can be stored in the reactant tanks, the product tank weight and volume are not included.

advantage of the dual-engine system (in which one engine is designed for dash speed, the other for cruise speed) relative to a single-engine system. Finally, the effects of the major vehicle and engine parameters on the weight/volume-limit criteria are illustrated for submersibles displacing 10,000 and 20 ft^3. Section IV summarizes results and conclusions.

II. Method of Analysis

The dash performance of the vehicle is the primary reference condition because it establishes the required maximum propulsive power P_d, which is the major determinant of engine weight. All items relating to power generation for propulsion and hotel load, the weights of which depend on power requirements only, are included in the engine weight W_e. All items whose weights depend on energy requirements, such as on-board reactants, tankage, tank insulation, and batteries, are included in the reactant weight W_r. The items V_e and V_r represent the volumes occupied by the items in W_e and W_r, respectively, including the space in the vehicle preempted by the presence of each of these items and all associated actions, even if that space is empty (e.g., area in the vehicle provided for servicing or for storing reaction products).

Let ωW be the weight permissible for the propulsion system, the reactants carried on board, and the electric power generation for the hotel load. The volume available in the vehicle for these items is designated θV. For a neutrally buoyant vehicle of total displacement weight W and volume V, we have for a weight-limited system

$$\omega W = W_e + W_r \tag{1}$$

and for a volume-limited system

$$\theta V = V_e + V_r \tag{2}$$

Equations (1) and (2) can be rewritten, respectively, as

$$\omega \rho_w V^* = W_e^* + W_{rd}^* t_d \tag{3}$$

and

$$\theta V^* = V_e^* + W_{rd}^* t_d / \rho_r \tag{4}$$

where

$$V^* = V/P_d \tag{5}$$

$$W_e^* = \frac{W_e}{P_d} = \frac{1}{P_d}\sum_i W_{e_i} \tag{6}$$

$$W_{rd}^* = (\mathrm{SRC})_d f \frac{P_s}{P_d} = (\mathrm{SRC})_d f\left(\frac{1}{\eta_p \eta_t} + \frac{P_h}{\eta_e P_d}\right) \tag{7}$$

$$\rho_e = W_e / V_e \tag{8}$$

$$\rho_r = W_r / V_r \tag{9}$$

$$V_e^* = \frac{V_e}{P_d} = \frac{1}{P_d}\sum_i \frac{W_{e_i}}{\rho_{e_i}} \tag{10}$$

$$\mathrm{SRC} = 2545/h\eta_{\mathrm{en}} \tag{11}$$

The quantity $\sum_i W_e$ is the sum of all elements included in the engine system such as the dash and cruise engines, gears, electric generators, shaft, and propeller. The quantity ρ_{e_i} is the density of the ith element, and allows for the space in the vehicle preempted by that element. The density ρ_r is equal to W_r divided by the total vehicle volume preempted by the reactants and all related items, such as tanks, insulation, and service space.

The value t_d is the time needed to expend all of the propellant if the vehicle were operated continuously at dash speed. Equations (3) and (4) each can be solved for t_d, giving, for the weight-limited case,

$$t_d = (\rho_w \omega V^* - W_e^*)(1/W_{rd}^*) \tag{12}$$

and, for the volume-limited case,

$$t_d = (\theta V^* - V_e^*)(\rho_r / W_{rd}^*) \tag{13}$$

If the inequality

$$(\rho_w \omega V^* - W_e^*) < (\theta V^* - V_e^*)\rho_r \tag{14}$$

holds, then the system is weight limited and Eq. (12) is used. If the inequality is not satisfied, then the system is volume limited and Eq. (13) is used. In the latter case, the vehicle must carry additional ballast for neutral buoyancy, the volume of which is neglected in the present equations.

The maximum cruise duration if all rectants on board were expanded at cruise speed is given by

$$t_c = t_d (P_d W_{rd}^* / P_c W_{rc}^*) \tag{15}$$

where

$$W_{rc}^* = (\mathrm{SRC})_c f(1/\eta_p \eta_t + P_h/\eta_e P_c) \tag{16}$$

The distances traversed are

$$X_d = U_d t_d \tag{17}$$

and

$$X_c = U_c t_c \tag{18}$$

For the weight-limited case, Eqs. (17) and (18) can also be written§

$$X_d = U_d (\rho_w \omega V^* - W_e^*) h\eta_{\mathrm{en}} / 2545 f(1/\eta_p \eta_t + P_h/\eta_e P_d) \tag{19}$$

and

$$X_c = U_c (\rho_w \omega V^* - W_e^*) h\eta_{\mathrm{en}} P_d / 2545 P_c f(1/\eta_p \eta_t + P_h/\eta_e P_c) \tag{20}$$

Similarly, for the volume-limited case, we may write

$$X_d = U_d (\theta V^* - V_e^*) \rho_r h\eta_{\mathrm{en}} / 2545 f(1/\eta_p \eta_t + P_h/\eta_e P_d) \tag{21}$$

and

$$X_c = U_c (\theta V^* - V_e^*) \rho_r h\eta_{\mathrm{en}} P_d / 2545 P_c f(1/\eta_p \eta_t + P_h/\eta_e P_c) \tag{22}$$

The quantities $\rho_w \omega V^*$ and θV^* depend on the vehicle design and the dash speed. The quantity W_e^* is substantially proportional to the weight per horsepower of the engine system chosen and, as a first approximation, can be assumed independent of the design dash speed. The engine efficiency depends on the operating power relative to the design power, and so the appropriate value corresponding to the speed condition under consideration should be used. The quantity f, which represents the ratio of the weight of the reactants plus tanks to the weight of the reactants alone, depends on both the size of the tanks and the physical characteristics of the reactants. For example, in exploratory studies of the performance of a vehicle of a given size with various reactants, it is sufficiently accurate to assume that f is a constant for

§Based on Eqs. (7, 11, 12, and 16).

noncryogenic reactants; a somewhat higher value should be used for cryogenic reactants.

When the propulsion system consists of an electric motor driven by a battery, then the specific battery energy in terms of pounds per horsepower-hour is substituted for specific reactant consumption (SRC) in Eqs. (7) and (16), with η_t taken as the product of the motor and transmission efficiencies. The quantity W_e represents the combined weight of the motor, gears, shaft, propeller, power conditioner, and controls.

When a fuel cell is used to generate electricity for the motor that drives the propeller, then the weight of the fuel cell also is included in W_e, and the weight of reactants consumed per unit of energy output from the fuel cell becomes the value of SRC in terms of lb/hphr in Eqs. (7) and (16). The value of η_t again is taken equal to the product of the electric motor and transmission efficiencies.

The cruise/dash distance tradeoffs available from a given propulsion system can be represented as illustrated in Fig. 1 by a straight line that passes through the value of X_d, computed from Eq. (19) or (21), on the ordinate; and X_c, computed from Eq. (20) or (22), for a chosen cruise speed on the abscissa. This line is the locus of all combinations of values of dash and cruise distances (for the chosen cruise speed) available from the specified system.

Figure 1 compares two hypothetical propulsion systems. One might represent a system designed to favor dash economy; the other, one in which a particularly light dash engine is used with a sacrifice in dash efficiency. Such a display immediately identifies the system with the longer cruise distance for a given dash distance requirement.

An advantage of the method represented by Eqs. (19-22) is that, by involving only major variables that separate vehicle parameters from engine parameters, it permits quick estimation of the effects of changes in these parameters on performance tradeoffs curves such as shown in Fig. 1. For a given vehicle design and design dash speed, $\rho_w \omega V^*$ is a constant. For a given engine design, W_e^* is a constant; it is substantially independent of vehicle design and design dash speed. Therefore, the effect on range of changes in the design of a vehicle with a given engine type is obtained by changing only $\rho_w \omega V^*$ and is proportional to $\rho_w \omega V^* - W_e^*$. The effect on range of changes in specific engine weight is obtained by adjusting W_e^*; again the range is proportional to $\rho_w \omega V^* - W_e^*$. Once the vehicle design, dash speed, and specific engine weight have been chosen, the quantity $\rho_w \omega V^* - W_e^*$ remains constant for all cruise speeds. Equations (19-22) indicate that range is proportional to engine efficiency η_{en} and the heating value of the reactants h.

Once the vehicle range as displayed, for example, by Fig. 1 has been determined for one set of vehicle and engine

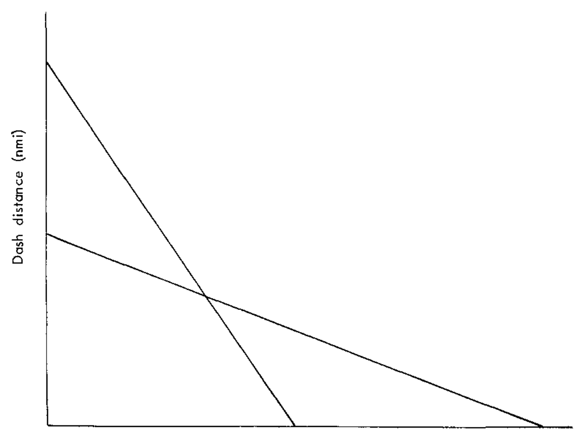

Fig. 1 Illustrative tradeoffs between cruise and dash distances for two propulsion systems.

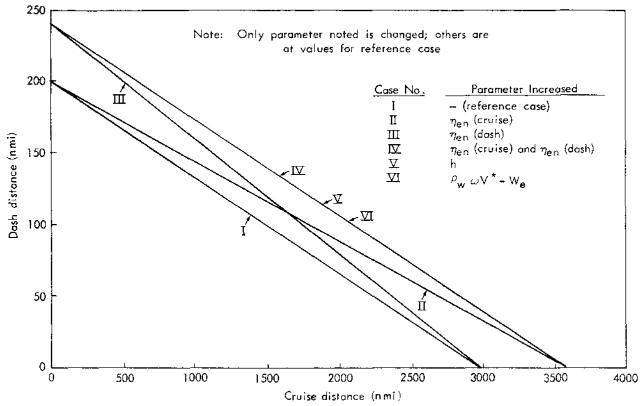

Fig. 2 Heuristic graph illustrating effect of 20% increase in parameters on range for specified vehicle, dash speed, and cruise speed.

parameters, adjustments to changes in these parameters can be established quickly. For example, a change in engine efficiency at dash speed causes a proportional change in the position of the end point of the performance line where it touches the ordinate axis. A change in engine efficiency at cruise speed similarly affects the position of the performance line where it touches the abscissa axis. A change in the quantity $\rho_w \omega V^* - W_e^*$ due to an altered vehicle design, dash speed, or specific engine weight moves the performance line parallel to itself a distance proportional to the the value of $\rho_w \omega V^* - W_e^*$. A different heating value for reactants, if it does not affect the engine cycle efficiency, moves the curve in Fig. 1 parallel to itself a distance proportional to h. Figure 2 illustrates these effects for a hypothetical vehicle. The parameters noted in the table in Fig. 2 are increased 20% relative to the values for the reference case.

The effects of various system parameters on dash and cruise distances can be obtained readily from Eqs. (19) and (20) or (21) and (22). In the following, we derive an equation that gives a more compact and generalized display of the effects of vehicle drag coefficient C_d and the weight factor ω. For a vehicle of volume V (given in cubic feet) and velocity U (given in knots), the vehicle drag power is

$$P = C_d \rho_w V^{2/3} (1.688U)^3 / 1100g, \text{ hp}$$

or

$$P = 0.00869 C_d V^{2/3} U^3, \text{ hp} \tag{23}$$

When the boundary layer lies mainly in the turbulent flow region, the drag coefficient based on volume displacement C_d varies as (Reynolds number)$^{-0.2}$. Most of the examples in the following will use a value of 0.025 for C_d. The effect on vehicle range of a variation in C_d from 0.020 to 0.030 will be displayed.

Equation (23) shows that, for a vehicle of given size and speed, the drag power is proportional to C_d; and therefore the term $\rho_w \omega V^*$ is proportional to ω / C_d. The engine weight is nearly proportional to P_d because the drag horsepower is large relative to the hotel load at dash speed; therefore, W_e^* is a constant with variation in ω and C_d. If the subscript o is used to represent a set of reference conditions (i.e., dash speed, ω, and C_d), then for the weight-limited case the effect of variations in ω and C_d on the cruise distance relative to that for the reference system may be obtained from Eq. (20), and is given by

$$\frac{X}{X_0} = \frac{\left[\left(\rho_w \omega_0 V^* \left(\frac{\omega C_{d0}}{\omega_0 C_d}\right) - W_e^*\right)\right]\left(\frac{1}{\eta_p \eta_t} + \frac{P_h}{\eta_e P_{c0}}\right)}{\left(\rho_w \omega_0 V^* - W_e^*\right)\left(\frac{1}{\eta_p \eta_t} + \frac{P_h C_{d0}}{\eta_e P_{c0} C_d}\right)} \tag{24}$$

In Eq. (24), the C_d values for cruise speed are assumed to change by the same ratio as the C_d values for dash speed. Let $b = W_e^*/\rho_w \omega_0 V^*$ and $a = 1 - b$; then from Eq. (23) we may write

$$b = 0.00869 W_e^* C_{d0} U_d^{\,3} / \omega_0 V^{1/3} \rho_w$$

The equation for X/X_0 then becomes

$$\frac{X}{X_0} = \frac{[(\omega/\omega_0)(C_{d0}/C_d) - b]}{a}$$

$$\frac{[(\eta_e/\eta_p\eta_t)(P_{c0}/P_h) + 1]}{[(\eta_e/\eta_p\eta_t)(P_{c0}/P_h) + C_{d0}/C_d]} \qquad (25)$$

Equation (25) also applies for the dash distance with P_{c0} replaced by P_{d0}. The derivation of Eq. (25) assumes that the system is weight limited, and that η_t, η_p, η_e, f, and SRC are independent of ω and C_d. If the system were volume limited, Eq. (25) would apply if b were defined as

$$b = V_e^*/\theta_0 V^* \qquad (26)$$

and again

$$a = 1 - b$$

The relative ranges given by Eqs. (24) and (25) are for a set of vehicles with the same volume and dash speed, differing only in the value of ω and C_d. The value of b, a constant in these equations, is determined by the reference parameters.

Table 1 Vehicle and engine parameters

Vehicle volume V, ft^3	10,000	
C_d	0.025[a]	
ω	0.25	
θ	0.50	
ρ_e, lb/ft^3	30.00	
ρ_r, lb/ft^3	64.00	
Dash speed U_d, knots	30.00	
$\rho_w \omega V^*$, lb/hp	73.90	
θV^*, ft^3/hp	2.31	
η_e	0.90	
$\eta_p \eta_t$	0.80	
f	1.20	
P_h, hotel load, hp	30.00	
	Engine A	Engine B
W_e^*, lb/hp	12.50	1.50
SRC, lb/hphr	1.50	3.00

[a] Although C_d varies with the Reynolds number, in this illustration it is assumed constant over the range of speeds considered.

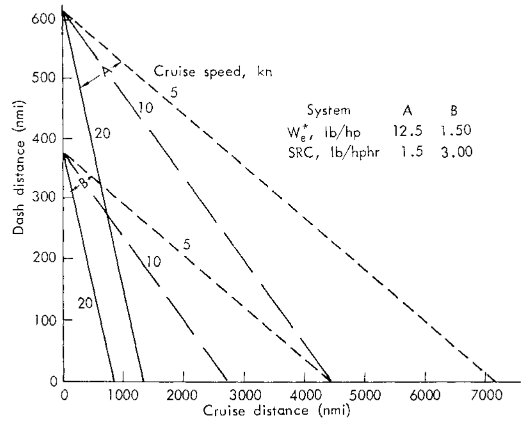

Fig. 3 Effect of trading specific engine weight for specific reactant consumption (vehicle displacement volume 10,000 ft^3, hotel power 30 hp, vehicle drag coefficient 0.025, dash speed 30 knots).

III. Application of Methodology

Large Manned Vehicles

In order to demonstrate the application of the methodology, we compare the range performances of a vehicle with a submerged displacement volume of 10,000 ft^3 using two alternative propulsion systems. System A is heavy, with a low specific reactant consumption; system B is light, but with a higher specific reactant consumption. Table 1 lists the parameters for the vehicle and both engines. These parameters describe hypothetical rather than specific systems in order to illustrate the application of the methodology. Substitution of the parameters from Table 1 into Eq. (14) reveals that the system is weight-limited, and so Eqs. (19) and (20) apply. Figure 3 shows the range tradeoffs between dash and various cruise speeds for the two propulsion systems.

Figure 3 demonstrates that system A gives the greater range for all of the cruise speeds shown, indicating the overwhelming importance of specific reactant consumption relative to specific engine weight for the systems characterized by Table 1. It is evident from Eqs. (19) and (20) that the range is inversely proportional to specific reactant consumption, and that the ranges in Fig. 3 can be adjusted to accommodate other values of SRC by applying this proportionality. (In other words, a 1% reduction in specific reactant consumption always provides a 1% increase in range.) The range is proportional to $\rho_w \omega V^* - W_e^*$. Therefore, when W_e^* is small relative to $\rho_w \omega V^*$ (as it is in the present example), then reduction in the value of W_e^* has little effect on range.

Table 2 summarizes the effects on cruise and dash ranges of a 1% reduction in specific engine weight for a broad range of

Table 2 Percent increase in range at both cruise and dash speeds for 1% reduction in specific engine weight for weight-limited systems

Displacement volume, ft^3	ω	C_d, dash	Specific engine weight,[a] dash, lb/hp	$\eta_p \eta_t$	Design dash speed U_d (knots)	Percent increase in range
10,000	0.25	0.020	10	0.80	30	0.21
10,000	0.50	0.020	10	0.80	30	0.094
10,000	0.25	0.025	10	0.80	30	0.27
10,000	0.50	0.025	10	0.80	30	0.12
10,000	0.25	0.025	10	0.80	35	0.52
10,000	0.25	0.025	10	0.80	40	1.02
10,000	0.25	0.025	10	0.80	50	7.62
20,000	0.25	0.025	10	0.80	30	0.20
5,000	0.25	0.025	10	0.80	30	0.37
10,000	0.25	0.025	1	0.80	30	0.02
10,000	0.25	0.030	10	0.80	30	0.34
10,000	0.50	0.030	10	0.80	30	0.15

[a] Engine weight divided by shaft power at 100% power.

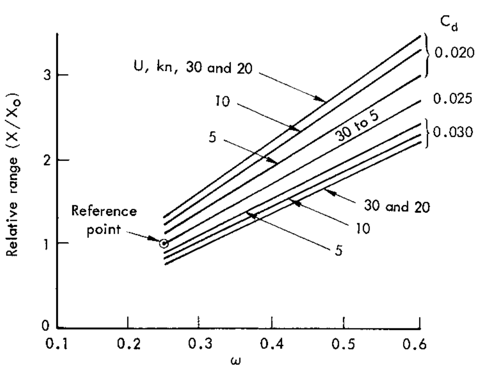

Fig. 4 Effect of variation in ω and C_d on range relative to range at same speed for reference system (system A, Fig. 3): $\omega_0 = 0.25$, $C_{d0} = 0.025$, $V = 10{,}000\ \mathrm{ft}^3$, $U_d = 30$ knots.

system parameters. These data indicate that at low design dash speeds (e.g., 30 knots) a 1% reduction in specific engine weight gives much less than a 1% increase in range. As design dash speed is increased, $\rho_w \omega V^*$ decreases. Finally, a point is reached (as Table 2 shows) where a decrease of 1% in specific engine weight can provide much more than a 1% increase in range, and specific engine weight can become the primary engine consideration.

Figure 4, computed from Eq. (25), shows the sensitivity of the range of system A of Fig. 3 to a change in the drag coefficient of the vehicle C_d, and the weight fraction of the vehicle available for the engine system and reactants ω. The ranges shown at the ends of the curves in Fig. 3 (for $C_{d0} = 0.025$ and $\omega_0 = 0.25$) are taken as the reference values in Fig. 4. At a value of $C_d = 0.025$, the relative range is, of course, independent of vehicle speed and extends linearly with increases in ω, as Eq. (25) shows.¶ The value of ω increases if the design submergence depth is decreased. It also can be increased by constructing the hull of materials having a higher strength per unit weight.

Range can be increased by decreasing C_d. According to Fig. 4, this effect increases with greater vehicle velocity. At a speed of 30 knots, decreasing C_d to 0.020 and doubling ω increases range by a factor of 2.9. At a speed of 5 knots, these same changes in C_d and ω increase range by a factor of 2.5.

The example displayed in Fig. 3 assumes that the SRC remains constant over the entire speed range. In practice, however, this condition is difficult to achieve on a single engine because of the great difference in power required at dash and cruise speeds. For example, in the case of Fig. 3, the engine power at a speed of 5 knots is 2% of the engine power at dash speed. A constant SRC over the entire speed range investigated is attainable with a dual-engine system. Therefore, Fig. 3 is applicable to a dual-engine system in which one engine is designed for high speeds (30 to 20 knots) and the other for low speeds (10 to 5 knots). The value of 12.50 lb/hp for W_e^* includes the weights of both engines.

The dash engine of system A (Table 1) in a single-engine system reasonably can be assumed to have an SRC of 1.50 lb/hphr at speeds of 30 and 20 knots, and SRC values of 2.0 and 4.4 at speeds of 10 and 5 knots, respectively.** It also is reasonable to assume that the single-engine system might weigh 5% less than the dual-engine system – i.e., the value of W_e^* would be 11.9 lb/hp. Figure 5 shows the vehicle ranges for this set of parameters designated as the single-engine system, and the vehicle ranges for system A of Fig. 3 designated as the dual-engine system. The dual-engine system shows a small loss in range (1%) at 30 and 20 knots, but a large gain at 10

¶Because b relates to the design dash speed (30 knots) and the volume of the reference vehicle (10,000 ft³), its value in Fig. 4 is constant (0.169).

**Based on curves showing the relative change of SRC with percent full load power for typical engines.

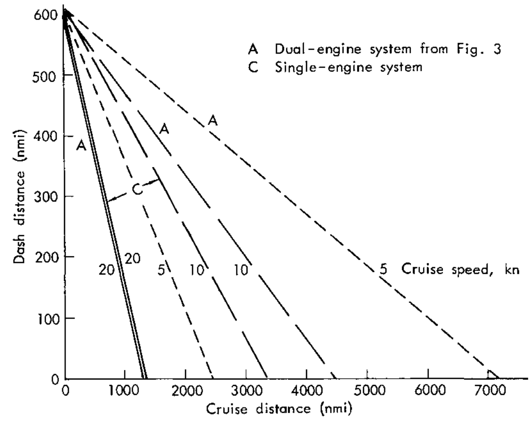

Fig. 5 Comparison of range of vehicles with single- and dual-engine systems (vehicle displacement volume 10,000 ft³, dash speed 30 knots).

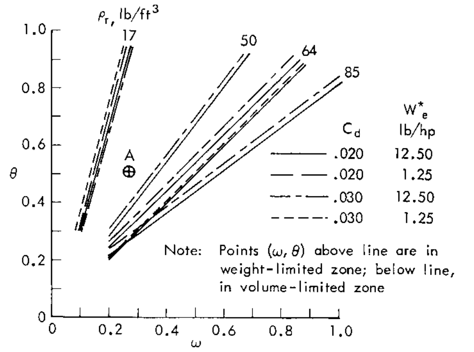

Fig. 6 Weight/volume-limit criteria for submarine with displacement volume 10,000 ft³, dash speed 30 knots, $\rho_e = 30\ \mathrm{lb/ft^3}$.

and 5 knots. The advantage of the dual-engine system increases as the required operating speed range is broadened.

The same reactant combination is used for each of the systems compared in Figs. 3-5. The relative ranges remain the same regardless of the reactant combination chosen because, as indicated by Eqs. (19) and (20), the ranges at dash and cruise speeds vary directly in proportion to the effective heating value of the reactant combination. For engines in which the heat of reaction is transferred from the reactants to the working fluid, the engine efficiencies η_{en} should be independent of the reactants employed. The effective heating value should allow for the heat removed from the system by the products of reaction at discharge temperature.

Equation (14) establishes whether a system is weight or volume limited. The boundary between the two zones can be determined by equating both sides of that equation, giving the expression

$$\theta = \left(\frac{\rho_w}{\rho_r}\right)\omega - \frac{W_e^*}{V^*}\left(\frac{I}{\rho_r} - \frac{I}{\rho_e}\right) \tag{27}$$

A plot of Eq. (27) for various values of ρ_r, C_d, and W_e^* appears as Fig. 6 for a submarine displacing 10,000 ft³ designed for a dash speed of 30 knots.

Point A in Fig. 6 represents the pair of values $\omega = 0.25$, $\theta = 0.50$, corresponding to the submarine design of Table 1. These values represent the weight and volume, respectively, left for propulsion system plus reactants after subtracting from the submarine displacement weight and volume, respectively, the weights and volumes preempted by the hull, fittings, miscellaneous equipment, crew, crew quarters, payload, and miscellaneous storage areas.

When the design values of ω and θ, as illustrated by point A in Fig. 6, lie above a line representing the parameters of the

Table 3 Densities of typical reactant combinations

Fuel	Oxidant	Fuel/oxidant stoichiometric ratio	Specific gravity Fuel			Density, stoichiometric mix, lb/ft^3	ρ_r [a] lb/ft^3
			Fuel	Oxidant	Stoichiometric mix		
Li	SF_6	0.380	0.466	1.34	0.844	55.1	50.9
Mg	O_2 [b]	1.52	1.74	0.89	1.26	78.8	72.7
Mg	SF_6	0.666	1.74	1.34	1.48	92.1	85.0
Al	O_2 [b]	1.12	2.70	0.890	1.38	86.1	79.4
H_2	O_2 [b]	0.126	0.070	0.890	0.385	24.0	17.0 [c]
$C_{10}H_{14}$	O_2 [b]	0.311	0.840	0.890	0.878	54.8	50.6

[a] $\rho_r = (1.2/1.3)$ density, where 1.2 allows for tankage weight and 1.3 allows for submarine volume preempted by the reactants for associated structures and maintenance space.
[b] Liquid oxygen.
[c] An allowance is made for the weight and volume of the tank containing the product water.

Table 4 Critical values of ω and θ for Fig. 6

W_e^* lb/hp	C_d	ω_c	θ_c
1.25	0.020	0.004	0.009
12.50	0.020	0.042	0.090
1.25	0.030	0.006	0.014
12.50	0.030	0.063	0.135

system, then the system is weight limited; when the point lies below the line, the system is volume limited. The positions of the lines in Fig. 6 depend mainly on the reactant density ρ_r. The slopes of the lines are equal to ρ_w/ρ_r. The design represented by point A thus is volume limited for a reactant density of 17 lb/ft^3 and is weight limited for the other densities shown on the figure. Table 3 shows values of ρ_r for several reactant combinations. For all combinations except $H_2 + O_2$, point A falls in the weight-limited zone. For the conditions represented by each line in Fig. 6, there is a specific value of ω and of θ at which the range falls to zero. These are called the critical values, designated ω_c and θ_c.

Equations (19-22) indicate that these critical values are given by

$$\omega_c = W_e^*/\rho_w V^* \tag{28}$$

and

$$\theta_c = V_e^*/V^* = (\rho_w/\rho_e)\omega_c \tag{29}$$

Table 4 lists the values of ω_c and θ_c for the conditions of Fig. 6. Point A and all lines in Fig. 6 lie above these critical values. The data in Fig. 6 relate to a design dash speed of 30 knots. Figure 7 shows the effect of varying the design dash speed for the same submarine. Each line was terminated at ω_c or θ_c, whichever was reached first.

Figure 7 demonstrates that, for the conditions listed and for a dash speed of 50 knots and drag coefficient of 0.03, point A is not a realistic possibility; the propulsion system weight and volume would exceed the available values. For a dash speed of 50 knots and a drag coefficient of 0.02, a design is possible but its range would be extremely small.

The upward movement of the lines with increases in U_d noted in Fig. 7 results because ρ_r is greater than ρ_e, as shown by Eq. (27). If a ρ_r of 17 lb/ft^3 were chosen, then the direction of the displacement of the lines with increases in U_d would be downward, as illustrated in the following section.

Small Unmanned Vehicles

Because much less space can be allowed for servicing the engine in small unmanned vehicles, the effective engine density ρ_e increases from about 30 lb/ft^3 for large vehicles to about 100 lb/ft^3 for small vehicles displacing about 20 ft^3. The factor f, which takes into account the reactant tank and insulation weight, increases because the smaller reactant tanks of the small vehicle have larger surface-area-to-volume ratios.

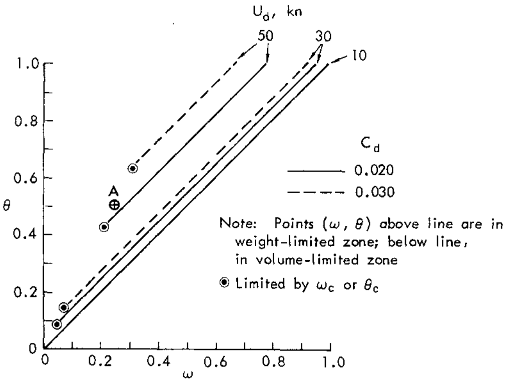

Fig. 7 Effect of design dash speed on weight/volume-limit criteria for submarine with displacement volume 10,000 ft^3, ρ_r = 64 lb/ft^3, ρ_e = 30 lb/ft^3, W_e^* = 12.5 lb/hp.

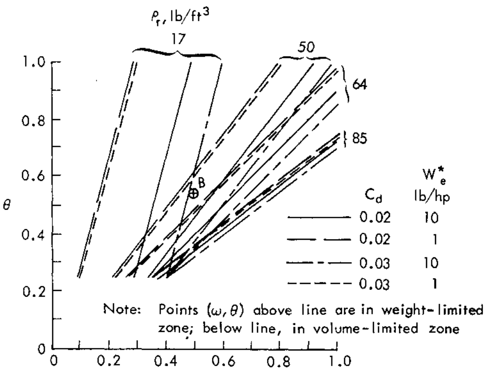

Fig. 8 Weight/volume-limit criteria for small submersible with V = 20 ft^3, U_d = 30 knots, ρ_e = 100 lb/ft^3.

Because Figs. 3-5 amply illustrate the method of comparing the range performances for the large vehicle of submersibles with various engines, we will not repeat this exercise for small vehicles.

The discussion of the small vehicle will be limited to the weight/volume-limit criteria that exhibit different characteristics than those of the large vehicle. These criteria are shown in Fig. 8 for a small submersible, displacing 20 ft^3 designed for a dash speed of 30 knots. Again, if the point (ω,θ) for a given vehicle falls above the line corresponding to the values of C_d and W_e^*, the system is weight limited; if below the line, the system is volume limited. The positions of the lines are more sensitive to the values of C_d and W_e^* for small than for large submersibles.

Illustrative point B at (ω,θ) of (0.50, 0.55) is typical of some small submersibles, such as torpedoes. It lies in the volume-limited zone for reactant combinations having a value for ρ_r of 17 lb/ft^3; and in the weight-limited zone for ρ_r of 64

Table 5 Critical values of ω and θ for Fig. 8

W_e^*, lb/hp	C_d	ω_c	θ_c
1	0.020	0.027	0.017
10	0.020	0.269	0.172
1	0.030	0.040	0.026
10	0.030	0.404	0.258

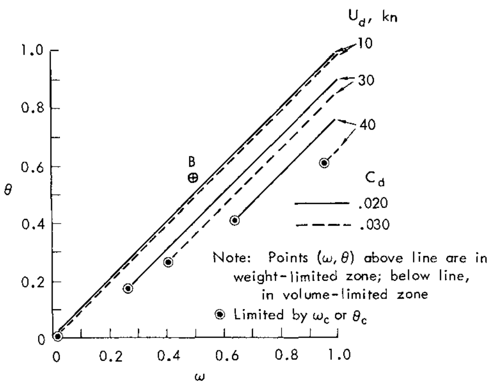

Fig. 9 Effect of design dash speed on weight/volume-limit criteria for small submersible with $V = 20$ ft^3, $\rho_e = 100$ lb/ft^3, $\rho_r = 64$ lb/ft^3.

and 85 lb/ft^3. For $\rho_r = 50$, point B lies in the volume-limited zone for $W_e^* = 1$, and it lies in the weight-limited zone for $W_e^* = 10$. The criteria for determining whether a system is weight or volume limited are more sensitive to reactant density for small submersibles than for large. Table 5 lists the critical values of ω and θ for Fig. 8. Point B in Fig. 8 lies above these critical values.

As previously noted in the discussion of Fig. 7 and Eq. (27), an increase in design dash speed causes a shift toward the weight-limited condition when ρ_r is less than ρ_e. That phenomenon is illustrated in Fig. 9, where ρ_r is 64 lb/ft^3 and ρ_e is 100 lb/ft^3.

IV. Concluding Remarks

Illustrative applications of the proposed method for evaluating the performances of chemically fueled underwater vehicles show that, for a submarine displacing 10,000 ft^3 designed for dash speeds of about 30 to 35 knots, a given percentage decrease in specific engine weight provides a much smaller percentage increase in range; whereas a given percentage decrease in specific fuel consumption at any given speed provides an equal percentage increase in range. At a design dash speed somewhat above 40 knots, a given percentage reduction in specific engine weight can provide a greater percentage increase in range. At high design dash speeds, therefore, the specific engine weight can become more important than reactant consumption.

With a single engine to cover the entire speed range, a vehicle designed for a dash speed of 30 knots suffers a considerable loss in efficiency at low cruise speeds, which is reflected as a great loss in cruise range. A dual-engine system, with one engine designed for high speed and the other for low speed, can provide great engine efficiency at all speeds and can give more attractive range performance, despite its somewhat greater weight.

Decreasing the vehicle drag coefficient C_d, and increasing the fraction of the gross vehicle weight available for engine and reactants ω, enhances the effects of each on relative range. Moreover, the benefits of these changes are magnified at higher speeds.

Whether a given system is weight or volume limited depends mainly on the values of the design dash speed, the net reactant density, the fraction of gross vehicle weight available for engine and reactants, and the fraction of the displacement volume available for engine and reactants. It depends to a lesser degree on the vehicle drag coefficient and specific engine weight. It is not affected by the specific engine reactant consumption.

For the representative vehicle displacing 10,000 ft^3 designed for a dash speed of 30 knots, the system is volume limited only for the reactant combination of liquid H_2 + liquid O_2, for which the effective reactant density ρ_r is in the vicinity of 17 lb/ft^3. For other reactant combinations investigated, the reactant densities are 2.5 to 4 times higher than that of liquid H_2 + liquid O_2, and the system will be weight limited.